## Capacitors

1. A student is modelling the decay of charge for a capacitor discharging through a resistor using the equation $\Delta Q$ $\Delta t=-0.2 Q$.
The student decides to use $\Delta \mathrm{t}=0.5 \mathrm{~s}$.
The initial charge on the capacitor is $1000 \mu \mathrm{C}$.
Part of the modelling spreadsheet from the student is shown below.

| $\mathbf{t} / \mathbf{s}$ | Charge $\mathbf{Q}$ left on <br> capacitor at time $\boldsymbol{t} / \boldsymbol{\mu} \mathbf{C}$ | Charge $\Delta \mathbf{Q}$ decaying in <br> the next $\mathbf{0 . 5} \mathbf{~} / \boldsymbol{\mu} \mathbf{C}$ |
| :---: | :---: | :---: |
| 0 | 1000 | 100 |
| 0.5 | 900 |  |
| 1.0 |  |  |
| 1.5 |  |  |

What is the value of Q in $\mu \mathrm{C}$ at $t=1.5 \mathrm{~s}$ ?
A 700
B 720
C 729
D 800

Your answer

2. The diagram below shows a circuit used to charge a capacitor.


The power supply has electromotive force (e.m.f.) 10 V and negligible internal resistance.
The capacitor has capacitance $C$ and the resistor has resistance $R$.
The switch is closed at time $t=0$.
The table below shows potential difference $V$ across the resistor at various values of time $t$.

| $\boldsymbol{V} / \mathbf{V}$ | 10 | 6.3 | 5.0 | 3.7 |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{t} / \mathbf{s}$ | 0 | 2.8 | 4.2 | 6.0 |

What is the product $C \times R$ for this circuit?
A 0 s
B $\quad 2.8 \mathrm{~s}$
C $\quad 4.2 \mathrm{~s}$
D $\quad 6.0 \mathrm{~s}$

Your answer $\square$

3 (a). A student designs an investigation to learn more about an old instrument called a hot wire ammeter. A fine resistance wire stretched between two retort stands sags when heated by the current being measured. This sag is converted into a reading on a non-linear scale.

A current-carrying wire is clamped at each end as shown in Fig. 2.1.


Fig. 2.1

The deflection $y$ at the centre of the wire is measured for various currents / in the wire.
It is suggested that $y$ and $I$ are related by the equation

$$
y=a l^{b}
$$

where $a$ and $b$ are constants. This equation can also be written as

$$
\lg y=\lg a+b \lg l
$$

A graph is plotted of Ig $y$ on the $y$-axis against Ig I on the $x$-axis. State expressions for the gradient and $y$-intercept in terms of $a$ and $b$.
gradient $=$ $\qquad$
$y$-intercept $=$ $\qquad$
(b). For different values of the current $I$, the vertical deflection $y$ is recorded. A table of results is shown with further columns giving values of $\lg \left(I / 10^{-2} \mathrm{~A}\right)$ and $\lg (\mathrm{y} / \mathrm{mm})$, including the absolute uncertainties.

| $\boldsymbol{/ / \mathbf { 1 0 } ^ { - \mathbf { 2 } } \mathbf { A }}$ | $\boldsymbol{y} / \mathbf{m m}$ | $\boldsymbol{\operatorname { l g }}\left(\boldsymbol{/} / \mathbf{1 0}^{-\mathbf{2}} \mathbf{A}\right)$ | $\boldsymbol{\operatorname { l g }}(\boldsymbol{y} / \mathbf{m m})$ |
| :---: | :---: | :---: | :---: |
| 50 | $2.6 \pm 0.2$ |  |  |
| 60 | $3.4 \pm 0.2$ | 1.78 | $0.53 \pm 0.03$ |
| 70 | $4.4 \pm 0.2$ | 1.85 | $0.64 \pm 0.02$ |
| 80 | $5.4 \pm 0.2$ | 1.90 | $0.73 \pm 0.02$ |
| 90 | $6.6 \pm 0.2$ | 1.95 | $0.82 \pm 0.01$ |
| 95 | $7.2 \pm 0.2$ | 1.98 | $0.86 \pm 0.01$ |

i. Complete the missing values in the table, including the absolute uncertainty for $\lg (y / \mathrm{mm})$.
ii. Fig. 2.2 shows the axes for a graph of $\lg (y / \mathrm{mm})$ on the $y$-axis against $\lg \left(/ / 10^{-2} \mathrm{~A}\right)$ on the $x$-axis. The last four points have been plotted including error bars for $\lg (y / m m)$. By plotting the two remaining points, complete the graph. Draw a line of best fit.


Fig. 2.2
(c).
i. Use the line of best fit through the data points in Fig. 2.2 to determine numerical values of $1 b$
$\qquad$
$b=$.

2 a.
$a=$
[2]
ii. Determine the absolute uncertainty in the value of $b$.
uncertainty in $b= \pm$

### 6.1 Capacitors

4 (a). The diagram below shows a circuit containing two capacitors which are both initially uncharged. The battery has e.m.f. $E$ and negligible internal resistance.

The switch $\mathbf{S}$ is first moved to position $\mathbf{A}$ until the capacitor of capacitance $C_{0}$ is fully charged.


The switch $\mathbf{S}$ is then moved to position $\mathbf{B}$. The initial charge stored by the capacitor of capacitance $C_{0}$ is shared between the two capacitors
The final reading on the voltmeter is $V$.
Show that $V=\frac{C_{0}}{C+C_{0}} E$.
(b). A student wants to determine the values of $E$ and $C_{0}$ by repeating the experiment above and measuring the potential difference (p.d.) $V$ for a selection of capacitors of capacitance $C$.
The student decides to plot a graph of $\frac{1}{V}$ against $C$.
i. Use the expression in (a) to show that the graph should be a straight line of gradient $\frac{1}{C_{0} E_{\text {and }}}$ and $y$-intercept $\frac{1}{E}$
ii. The data points, error bars and the line of best fit drawn by the student are shown in the graph below.


The gradient of the line of best fit is $51 \mathrm{~V}^{-1} \mathrm{~F}^{-1}$. The value of $E$ is 9.1 V .
Determine the value of $C_{0}$ in millifarads ( mF ). Write your answer to 2 significant figures.

$$
C_{0}=.
$$

$\qquad$
iii. Draw on the graph a straight line of worst fit.

Use this line to determine the absolute uncertainty in your value of $C_{0}$. Write your answer to an appropriate number of significant figures.

### 6.1 Capacitors

(c). The experiment is repeated with a resistor of resistance $10 \mathrm{k} \Omega$ placed in series between $\mathbf{S}$ and the capacitor of capacitance $C$.

State with a reason what effect, if any, this would have on the experiment.
$\qquad$
5. The diagram below shows a circuit connected by a student.


The capacitance of each capacitor is 300 pF .
What is the total capacitance between points $\mathbf{R}$ and $\mathbf{S}$ ?

A $\quad 75 \mathrm{pF}$
B $\quad 230 \mathrm{pF}$
C $\quad 400 \mathrm{pF}$
D $\quad 1200 \mathrm{pF}$

Your answer
6. A capacitor is charged through a resistor.


The cell has e.m.f. 1.50 V and negligible internal resistance.
The capacitor is initially uncharged. The time constant of the circuit is 100 s .
The switch is closed at time $t=0$.
What is the potential difference across the capacitor at time $t=200 \mathrm{~s}$ ?
A $\quad 0.20 \mathrm{~V}$
B $\quad 0.55 \mathrm{~V}$
C 0.95 V
D $\quad 1.30 \mathrm{~V}$

Your answer $\square$
7. A capacitor discharges through a resistor.

At time $t=0$ the potential difference $V$ across the capacitor is $V_{0}$.
At time $t=2.0 \mathrm{~s}, V=0.90 \mathrm{~V}$.
Which statement is not correct?

A At $t=4.0 \mathrm{~s}, V=0.81 \mathrm{~V}_{0}$.
B The capacitor is fully discharged after $t=10 \mathrm{~s}$.
C The potential difference across the resistor is the same as that for the capacitor.
D The potential difference $V$ decreases exponentially with time $t$.

Your answer

8. Two isolated parallel capacitor plates have an equal and opposite charge.

The separation between the plates is doubled.
The charge on each plate remains the same but the potential difference between the plates doubles.


Which statement is correct?

A The capacitance of the capacitor doubles.
B The energy stored by the capacitor is halved.
C The permittivity of free space doubles.
D The electric field strength between the plates remains the same.

Your answer

9. A capacitor discharges through a resistor. At time $t=0$, the charge stored by the capacitor is $600 \mu \mathrm{C}$. The capacitor loses $5.0 \%$ of its charge every second.

What is the charge left on the capacitor at time $t=4.0 \mathrm{~s}$ ?

A $\quad 111 \mu \mathrm{C}$
B $\quad 120 \mu \mathrm{C}$
C $\quad 480 \mu \mathrm{C}$
D $\quad 489 \mu \mathrm{C}$

10. A capacitor consists of two parallel plates separated by air. The capacitor is connected across a d.c. supply. The charged capacitor is then disconnected and the separation between the plates is doubled.

Which statement is correct about the charge stored by the capacitor?
A The charge is the same.
B The charge doubles.
C The charge halves.
D The charge quarters.

Your answer $\square$
11. The graph below shows the variation of potential difference $V$ with charge $Q$ for a capacitor.


Which row is correct for the gradient of the graph and the area under the graph?

|  | Gradient of graph | Area under the graph |
| :--- | :--- | :--- |
| A | capacitance $^{-1}$ | work done |
| B | capacitance $^{-1}$ | permittivity |
| D | capacitance | power |
|  | capacitance | energy |

Your answer $\square$
12. A capacitor is discharged through a resistor.


The capacitor is fully charged at time $t=0$. The time constant of the circuit is 10 s . The switch is closed at time $t=$ 0 . The current in the resistor is $I$.

Which row is correct?

|  | Current $\boldsymbol{I}$ at $\boldsymbol{t}=\mathbf{0}$ | Current $\boldsymbol{I}$ at $\boldsymbol{t}=\mathbf{1 0} \mathbf{s}$ |
| :---: | :---: | :---: |
| $\mathbf{A}$ | maximum | 0 |
| $\mathbf{B}$ | maximum | $37 \%$ of the current at $t=0$ |
| $\mathbf{C}$ | 0 | $63 \%$ of the current at $t=\infty$ |
| $\mathbf{D}$ | 0 | $37 \%$ of the current at $t=\infty$ |

Your answer $\square$
13. A capacitor is charged through a resistor.


The cell has electromotive force (e.m.f.) 1.50 V and negligible internal resistance.
The time constant of the circuit is 10 s . The switch is closed at time $t=0$. At time $t$, the potential difference across the resistor is 0.60 V .

Which expression is correct?

A $\quad 0.60=1.50 \mathrm{e}^{-0.10 t}$
B $\quad 0.90=1.50 \mathrm{e}^{-0.10 t}$
C $\quad 0.60=1.50 \mathrm{e}^{-10 t}$
D $0.60=1.50\left(1-\mathrm{e}^{-10 t}\right)$

Your answer

14. Four capacitors of capacitance $10 \mu \mathrm{~F}, 20 \mu \mathrm{~F}, 30 \mu \mathrm{~F}$ and $40 \mu \mathrm{~F}$ are connected in series to a battery.

Which capacitor has the largest potential difference across it?

A $\quad 10 \mu \mathrm{~F}$ capacitor
B $\quad 20 \mu \mathrm{~F}$ capacitor
C $\quad 30 \mu \mathrm{~F}$ capacitor
D $\quad 40 \mu \mathrm{~F}$ capacitor

Your answer $\square$

15 (a). A student is investigating how the discharge of a capacitor through a resistor depends on the resistance of the resistor.
The equipment is set up as shown in Fig. 3.1.


Fig. 3.1

The student charges the capacitor of capacitance $C$ and then discharges it through a resistor of resistance $R$ using switch $\mathbf{S}$. After a time $t=15.0 \mathrm{~s}$ the student records the potential difference $V$ across the capacitor. The student repeats this procedure for different values of $R$.

It is suggested that $V$ and $R$ are related by the equation

$$
V=V_{0} \mathrm{e}^{-\frac{t}{C R}}
$$

where $V_{0}$ is the initial potential difference across the capacitor and $t$ is the time over which the capacitor has discharged.

The student decides to plot a graph of $\ln (V / V)$ on the $y$-axis against $\bar{R}$ on the $x$-axis to obtain a straight line graph. Show that the magnitude of the gradient is equal to $\frac{15}{C}$
(b). Values of $R$ and $V$ at $t=15.0 \mathrm{~s}$ are given in the table below.

| $R / k \Omega$ | $V / V$ | $\left(\frac{1}{R}\right) / 10^{-6} \Omega^{-1}$ | $\ln (V / V)$ |
| :---: | :---: | :---: | :---: |
| 56 | $3.0 \pm 0.2$ | 18 |  |
| 68 | $3.7 \pm 0.2$ | 15 | $1.31 \pm 0.06$ |
| 100 | $5.0 \pm 0.2$ | 10 | $1.61 \pm 0.04$ |
| 150 | $6.4 \pm 0.2$ | 6.7 | $1.86 \pm 0.03$ |
| 220 | $7.3 \pm 0.2$ | 4.5 | $1.99 \pm 0.03$ |
| 330 | $8.1 \pm 0.2$ | 3.0 | $2.09 \pm 0.03$ |

i. Complete the missing value of $\ln (V / V)$ and its absolute uncertainty in the table above
ii. Use the data to complete the graph of Fig. 3.2. Four of the six points have been plotted for you.


Fig. 3.2

### 6.1 Capacitors

iii. Use the graph to determine a value for $C$. Include the absolute uncertainty and an appropriate unit in your answer.
$C=$ $\qquad$ ..$\pm$. $\qquad$ unit
(c). Determine the value of $R$, in $\mathrm{k} \Omega$, for which the capacitor discharges to $10 \%$ of its original potential difference in 15.0 s. Show your working.

$$
R=.
$$

$\mathrm{k} \Omega$ [2]
16. Fig. 20.1 shows a capacitor and a switch connected in series to a cell.


Fig. 20.1
The switch $\mathbf{S}$ is closed.
Describe and explain how the capacitor plates $\mathbf{A}$ and $\mathbf{B}$ acquire opposite charges.
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$\qquad$
$\qquad$
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$\qquad$
$\qquad$
17. A nucleus of hydrogen-3 $\left({ }_{1}^{3} \mathrm{H}\right)$ is unstable and it emits a beta-minus particle (electron).

The emitted beta-minus particle enters a region of uniform magnetic field.
Fig. 22.1 shows the path of the particle before it enters the magnetic field.


Fig. 22.1
The direction of the magnetic field is into the plane of the paper.
Describe and explain the path of the particle in the magnetic field.
$\qquad$
$\qquad$
$\qquad$

## [2]

18. A student wishes to determine the permittivity $\varepsilon$ of paper using a capacitor made in the laboratory.

The capacitor consists of two large parallel aluminium plates separated by a very thin sheet of paper.
The capacitor is initially charged to a potential difference $V_{0}$ using a battery. The capacitor is then discharged through a fixed resistor of resistance $1.0 \mathrm{M} \Omega$.

The potential difference $V$ across the capacitor after a time $t$ is recorded by a data-logger. The student uses the data to draw the $\ln V$ against $t$ graph shown in Fig. 22.


Fig. 22

Show that the magnitude of the gradient of the line shown in Fig. 22 is equal to

$$
\frac{1}{C R}
$$

where $C$ is the capacitance of the capacitor and $R$ is the resistance of the resistor.

### 6.1 Capacitors

19. Fig. 20.1 shows a capacitor connected to a power supply.


Fig. 20.1
The capacitor consists of two parallel metal plates separated by air.
The switch is closed to charge the capacitor.
The switch is then opened and the separation between the charged plates is doubled.
State and explain what happens to the energy stored by the capacitor.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

20. Fig. 20.3 shows a capacitor-resistor circuit.


Fig. 20.3
Describe how the time constant of this circuit can be determined experimentally in the laboratory.
$\qquad$
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$\qquad$
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$\qquad$

### 6.1 Capacitors

21 (a). A capacitor of capacitance $C$ is connected across a strip of conductive paper.


The switch is moved from $\mathbf{X}$ to $\mathbf{Y}$, and the time $t$ for the potential difference across the capacitor to halve is measured.

The time $t$ is given by the expression
$t=(\mathrm{C} k \ln 2) \times \mathrm{L}$
where k is the resistance of the conductive paper per unit length and $L$ is the length of the conductive paper.
The value of C is $1.2 \times 10^{-3} \mathrm{~F}$.
In an experiment, $L$ is changed and $t$ measured.
The data points are plotted on a $t$ against $L$ grid as shown below.


Draw a straight line of best fit through the data points, and use the gradient of this line to determine $k$.
(b). The diagram below shows a circuit to charge a capacitor.


The electromotive force (e.m.f.) $E$ of the cell is 1.48 V and it has negligible internal resistance. The resistance of the resistor is $120 \mathrm{k} \Omega$ and the capacitance of the capacitor is $2000 \mu \mathrm{~F}$. At time $t=0$ the capacitor is uncharged.
The switch is closed at time $t=0$.
Calculate the time $t$ when the potential difference across the capacitor is 1.00 V .

$$
t=
$$

22 (a). A capacitor of capacitance 7.2 pF consists of two parallel metal plates separated by an insulator of thickness
1.2 mm . The area of overlap between the plates is $4.0 \times 10^{-4} \mathrm{~m}^{2}$. Calculate the permittivity of the insulator between the capacitor plates.
$\qquad$ $\mathrm{F} \mathrm{m}^{-1}[2]$
(b). Fig. 21 shows a circuit.


Fig. 21
The capacitance of each capacitor is $1000 \mu \mathrm{~F}$. The resistance of the resistor is $10 \mathrm{k} \Omega$. The cell has e.m.f. 1.5 V and negligible internal resistance.
i. Calculate the total capacitance $C$ in the circuit.

$$
C=
$$

### 6.1 Capacitors

ii. The switch $\mathbf{S}$ is closed at time $t=0$. There is zero potential difference across the capacitors at $t=0$. Calculate the potential difference $V$ across the resistor at time $t=12 \mathrm{~s}$.

$$
V=.
$$

23. Fig. 20.2 shows an arrangement of capacitors connected to a battery.


Fig. 20.2
The e.m.f. of the battery is 12 V .
Calculate the total energy $E$ stored by the capacitors in this circuit.

### 6.1 Capacitors

24. Fig. 22.1 shows two horizontal metal plates in a vacuum.


Fig. 22.1

The plates are connected to a power supply. The potential difference $V$ between the plates is constant. The magnitude of the charge on each plate is $Q$. The separation between the plates is $d$.

Fig. 22.2 shows the variation with $d$ of the charge $Q$ on the positive plate.


Fig. 22.2
i. Use Fig. 22.2 to propose and carry out a test to show that $Q$ is inversely proportional to $d$.

Test proposed:

Working:
ii. Use capacitor equations to show that $Q$ is inversely proportional to $d$.

25 (a). This question is about investigating the charging and discharging of capacitors.
Two students are given the circuit shown in Fig. 6.1. It consists of two resistors and two uncharged capacitors, a 10 V supply and a two-way switch $\mathbf{S}$.


Fig. 6.1

The first student is asked to investigate the charging of the capacitor $\mathbf{C}_{1}$ when $\mathbf{S}$ is connected to $\mathbf{A}$. She selects an ammeter of range 0 to $100 \mu \mathrm{~A}$ reading to $2 \mu \mathrm{~A}$ and a stopwatch reading to 0.1 s .

Discuss whether she has made a sensible choice of equipment for this experiment.
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$\qquad$
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$\qquad$
(b). A student is asked to investigate the change of potential difference (voltage) $V$ with time $t$ across each capacitor from the instant that $\mathbf{S}$ is moved from $\mathbf{A}$ to $\mathbf{B}$.
i. Explain why the final potential difference across each capacitor is 5.0 V .
$\qquad$
$\qquad$
$\qquad$
ii. Predict the outcome of the experiment by sketching two graphs on Fig. 6.2 to display the results that the student should obtain for each capacitor. Label them $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$.


Fig. 6.2
26. A student wishes to determine the permittivity $\varepsilon$ of paper using a capacitor made in the laboratory.

The capacitor consists of two large parallel aluminium plates separated by a very thin sheet of paper.
The capacitor is initially charged to a potential difference $V_{0}$ using a battery. The capacitor is then discharged through a fixed resistor of resistance $1.0 \mathrm{M} \Omega$.

The potential difference $V$ across the capacitor after a time $t$ is recorded by a data-logger. The student uses the data to draw the $\ln V$ against $t$ graph shown in Fig. 22.


Fig. 22

### 6.1 Capacitors

Use Fig. 22 to determine the capacitance $C$ of the capacitor. Describe how the student can then use this value of $C$ to determine a value for $\varepsilon$.
In your description, mention any additional measurements required on the capacitor.
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27. A student is carrying out an experiment in the laboratory to determine the capacitance $C$ of a capacitor. Fig. 20.2 shows the circuit used by the student.


Fig. 20.2
The switch $\mathbf{S}$ is first connected to $\mathbf{X}$ to charge the capacitor. The switch is then moved to $\mathbf{Y}$ at time $t=0$. The time $T$ taken for the potential difference $V$ across the capacitor to halve is determined for different values of resistance $R$.
i. Fig. 20.3 shows the graph of $T$ against $R$ as plotted by the student.


Fig. 20.3
1 Draw a straight line of best fit.

2 Use $V=V_{0} \mathrm{e}^{-t C R}$ to show that $T=-\ln (0.5) C R$.

3 Determine a value for the capacitance $C$.

$$
C=
$$

ii. Describe, without doing any calculations, how you can use Fig. 20.3 to determine the percentage uncertainty in $C$.
$\qquad$
$\qquad$
$\qquad$
28. Fig. 3.1 shows the design of a 'mechanical' torch.


Fig. 3.1
There is no battery in the torch. Instead, when the torch is inverted, the magnet falls a short vertical distance $h$ through the coil of wire, as shown in Fig. 3.2. This induces an electromotive force (e.m.f.) across the ends of the coil. The e.m.f. is used to store charge in a capacitor, which lights a light-emitting diode (LED) when it discharges.


Fig. 3.2

### 6.1 Capacitors

Fig. 3.3 shows the variation with time of the e.m.f. generated as the magnet falls the distance $h$.


Fig. 3.3

When the torch is inverted, the pulses of e.m.f. shown in Fig. 3.3 cause a capacitor of capacitance 0.12 F to become charged.
Each positive and each negative pulse adds $9.0 \times 10^{-3} \mathrm{C}$ to the charge stored in the capacitor.
i. The torch is inverted 80 times.

Calculate the total energy stored in the capacitor.
total energy $=$
ii. When the torch is switched on, the energy stored in the capacitor lights a 50 mW LED. Estimate the time for which the LED lights.
time $=$

